ANALYSIS OF HIGH RESOLUTION 3D TRAJECTORIES FOR GEO-REFERENCING PURPOSES

Corinna HARMENING, Jens-André PAFFENHOLZ, and Hamza ALKHATIB, Germany

Key words: least-squares collocation, Kalman filtering, kinematic GNSS trajectories

SUMMARY

The data acquisition of phenomena in Geodesy is typically performed in a discrete manner, often with respect to time. If data points are required for a specific time without original measured data, interpolation methods can be applied to obtain additional data points by means of measured data. Different methods can be used to solve the interpolation task.

This paper focuses on the assessment of a prediction approach with consideration of stochastic information. It is known that adjacent observations in time series have more mutual influence than far observations have to each other. While correlations between near observations are expected to be significantly large, their values tend to descend for far spatial distances.

The application of least-squares collocation on an interpolation problem arising from an approach of direct geo-referencing is presented in this paper. Geo-referencing of 3D point clouds generally means a procedure to estimate the relative transformation parameters between the local-defined laser scanner coordinate system and a global or absolute coordinate system.

The most important issue in collocation approaches is the estimation of an appropriate covariance function. This topic will be discussed in detail. The results of the collocation will be compared with Kalman filter results. First, both comparisons will be shown for the 3D trajectory itself. Second, the impact of the different approaches is discussed with respect to their significance for the previously introduced application: the direct geo-referencing of 3D point clouds acquired by stationary laser scanner.

ANALYSIS OF HIGH RESOLUTION 3D TRAJECTORIES FOR GEO-REFERENCING PURPOSES

Corinna HARMENING, Jens-André PAFFENHOLZ, and Hamza ALKHATIB, Germany

1. INTRODUCTION

The data acquisition of phenomena in Geodesy is typically performed in a discrete manner often with respect to time. If data points are required for a specific time without original measured data, interpolation methods can be applied to obtain additional data points by means of measured data. Different methods can be used to solve the interpolation task. Usually a functional relationship between the known data points is established and applied to determine the needed values. The common approach is the linear interpolation. In order to achieve better approximation higher-ordered polynomials can be used. As these higher-ordered polynomials can lead to oscillations, the interpolation by means of splines can be used instead (Niemeier, 2008). Another possibility to solve such interpolation problems is least-squares collocation (LSC). This approach differs from the aforementioned ones as follows: On the one hand, stochastic relationships are taken into account, on the other hand the influence of measurement noise is reduced. For that reason LSC normally leads to better results than the interpolation by polynomials or splines.

This paper deals with the application of LSC on an interpolation problem arising from an approach of direct geo-referencing. Geo-referencing of 3D point clouds generally means a procedure to estimate the relative transformation parameters between the local-defined laser scanner coordinate system and a global or absolute coordinate system. In case of using additional sensors, such as GNSS equipment and inclinometer, to estimate the laser scanner’s position and orientation, the geo-referencing is termed direct geo-referencing. A multi-sensor system (MSS) composed of a terrestrial laser scanner and eccentrically mounted GNSS antennas is applicable to provide the required transformation parameters. Therefore, the circular 3D trajectory of the GNSS antenna reference point (ARP) is evaluated in Kalman filter approach to simultaneously obtain the laser scanner’s position as well as orientation. The circular 3D trajectory of the ARP results from the laser scanner's rotation about its vertical axis (Paffenholz et al., 2010).

The above-mentioned approach is originally designed to handle all observations at the simultaneous time which demands an interpolation in case of different data acquisition rates. In the current MSS realisation, both sensors (laser scanner and GNSS equipment) operate with different acquisition rates. In other words, a synchronisation of both data is required. Up to now the default interpolation is performed in a linear manner (cf. Figure 1 left). An expected better result can be achieved by means of LSC (cf. Figure 1 right). The reason for that is, the prediction in LSC framework considers stochastic relationships among the data and the filtering in LSC framework decreases the influence of the measurement noise.

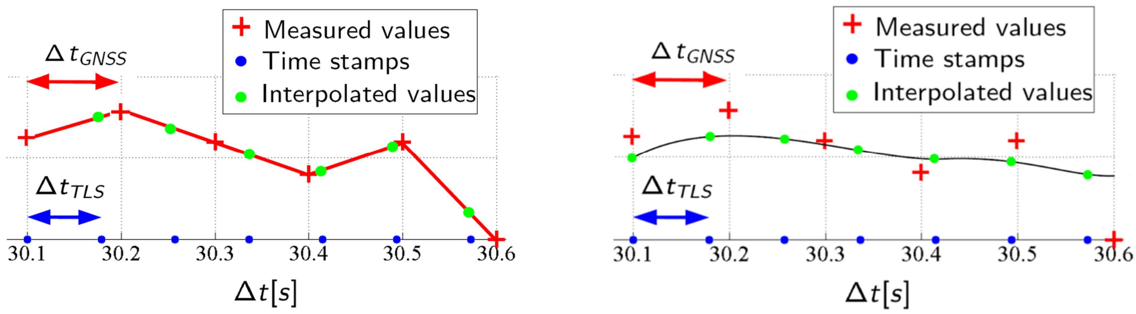

Figure 1: Approximation of GNSS positions with respect to scan profiles by linear interpolation (left) and by means of LSC (right).

Therefore, this paper deals with the development of an appropriate collocation approach to solve the interpolation task in the frame of the outlined direct geo-referencing approach.

The paper is organized as follows. Section 2 familiarizes the reader with the theory of leas-squares collocation. In particular the topics functional and stochastic model, derivation of covariance functions and the solution of the least-squares collocation by means of Gauss-Helmert model (GHM) are addressed. In Section 3 a collocation approach for prediction and filtering of 3D trajectories for a direct geo-referencing is proposed. The focus here is on the trend modelling, the determination of appropriate covariance functions as well as the solution by estimation as well as prediction and filtering of high resolution 3D GNSS trajectories. Finally, Section 4 summarizes the results and discusses future work.

2. BACKGROUND OF LEAST-SQUARES COLLOCATION

The approach of LSC has been introduced by Moritz (1962) in the field of geodesy. The approach is joining the three main tasks: parameter estimation, filtering and prediction (Moritz, 1980). Although LSC was primarily used to determine the gravity field, it is meanwhile used in many other fields of geodesy, for example to determine satellite orbits (Moritz, 1980) or to predict atmospheric fields (Teunissen, 2006).

2.1 Functional and stochastic model

Contrary to the classical Gauss-Markov model (GMM) (Koch, 1999, pp. 153 ff.), the $n \times 1$ vector of residuals $\mathbf{v}$ is defined in LSC by

$$\mathbf{v} = \mathbf{Ax} + \mathbf{Rs} - \mathbf{l}, \tag{1}$$

where $\mathbf{l}$ is the $n \times 1$ vector of observations, $\mathbf{x}$ is the $u \times 1$ unknown parameter vector, $\mathbf{A}$ is a known $n \times u$ design matrix. $\mathbf{R}$ denotes an $n \times (r - n)$ matrix of known coefficients and $\mathbf{s}$ is the $(r - n) \times 1$ signal vector.

In the model given in Eq. (1) the sum of $\mathbf{l}$ and $\mathbf{v}$ is equal to the trend, the regular systematic part $\mathbf{Ax}$, and an additional stochastic irregular systematic part $\mathbf{Rs}$, known as signal, see Koch (1999, pp. 221 ff.).

The purpose of LSC is the separation of the mentioned parts: By means of adjustment the optimal trend parameters are estimated, while the filtering reduces the influence of the measurement noise. The prediction is used to determine trend and signal between the data points. Whereas the trend can be predicted by means of the estimated trend parameters, the prediction of the signal requires variance-covariance matrices (VCMs). These matrices are set up based on the stochastic relationships of adjacent signal values. Assuming that noise and signal are normally distributed and mutually independent, the stochastic model can be written as (Welsch et al., 2000)

$$V\begin{pmatrix}\mathbf{v}\\ \mathbf{s}\\ \mathbf{s}'\end{pmatrix} = \begin{bmatrix}\mathbf{\Sigma_{vv}} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{\Sigma_{ss}} & \mathbf{\Sigma_{ss'}}\\ \mathbf{0} & \mathbf{\Sigma_{s's}} & \mathbf{\Sigma_{s's'}}\end{bmatrix}. \qquad (2)$$

In Eq. (2) $\mathbf{\Sigma_{vv}}$ and $\mathbf{\Sigma_{ss}}$, and $\mathbf{\Sigma_{s's'}}$ denote the VCMs of the noise, the signal and the predicted signal $\mathbf{s}'$, respectively. $\mathbf{\Sigma_{vv}}$ can be set up according to the VCM of observations. The main problem of set up the VCM given in Eq. (2) is the lower block dealing with the VCMs of $\mathbf{s}$ and $\mathbf{s}'$. This problem can be solved by covariance functions which can be derived based on observed data.

2.2 Derivation of covariance functions

Covariance functions are used in time series analysis to model stochastic relationships between one or more time series (Welsch et al., 2000). In order to estimate these functions, the underlying stochastic process has to fulfil the conditions of stationarity and ergodicity (Neuner, 2008). As these two conditions are not fulfilled if the time series contains a deterministic trend, this trend has to be removed. The corresponding trend function can be estimated by means of regression analysis.

The empirical auto-covariance function $C(d)$ can be computed by means of the values x_i, with $i = 1 \dots n$. x_i denotes the difference between the original measurements and the pre-estimated trend. $C(d)$ can be expressed as a function of the distance d, where d can be in the simple case of homogenous, regular time series the difference of multiple time steps,

$$C(d) = \frac{1}{n-d-1}\sum_{i=1}^{n-d}(x_i - \bar{x})(x_{i+d} - \bar{x}). \qquad (3)$$

In Eq. (3) $\bar{x}$ denotes the mean value of the time series and n is the number of measured values. The distance d is defined for time differences from 0 to $m = n/10$ due to the computation accuracy of the above-mentioned function (Welsch et al., 2000, p. 323). It is worth noting, that for $d = 0$ Eq. (3) yields the empirical variance. In order to allow better comparison of different auto-covariance functions, they are normalized by $C(0)$. The result of this normalization is the empirical auto-correlation function

$$K(d) = \frac{C(d)}{C(0)}. \qquad (4)$$

To obtain stochastic relationships between two different time series, cross-covariance functions $C_{xy}(d)$ and cross-correlation functions $K_{xy}(d)$, respectively, are used

$$C_{xy}(d) = \frac{1}{n-d-1}\sum_{i=1}^{n-d}(x_i - \bar{x})(y_{i+d} - \bar{y})\,, \tag{5}$$

$$K_{xy}(d) = \frac{C_{xy}(d)}{\sqrt{C_x(0)\cdot C_y(0)}}. \tag{6}$$

Those empirical functions provide the basis for the determination of analytical correlation functions $\widehat{K}(d)$ and $\widehat{K}_{xy}(d)$, which are estimated by the values of the empirical functions using regression analysis. By means of these analytical functions the VCM $\mathbf{\Sigma}$ and respective cofactor matrix $\mathbf{Q}$ can be computed as follows

$$\mathbf{\Sigma} = C(0)\cdot\ \mathbf{Q} = C(0)\cdot\begin{bmatrix} 1 & \widehat{K}(1) & \widehat{K}(2) & \cdots & \cdots & \cdots \\ \widehat{K}(1) & 1 & \widehat{K}(1) & \ddots & \ddots & \vdots \\ \widehat{K}(2) & \widehat{K}(1) & 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \widehat{K}(1) \\ \vdots & \ddots & \ddots & \ddots & \widehat{K}(1) & 1 \end{bmatrix}. \tag{7}$$

The properties of the VCM restrict the choice of the functions, which can be used to build the VCMs given in Eq. (2). On the one hand, the functions have to be positive definite, on the other hand the correlations which are calculated by means of these functions have to take values from the interval $[-1, 1]$.

2.3 Least-squares collocation by means of Gauss-Helmert model

The functional model in Eq. (1) is extend with the predicted signal $\mathbf{s}'$ due to the fact that both $\mathbf{s}'$ and $\mathbf{s}$ are mutually dependent according to Eq. (2). This results in (Welsch et al., 2000)

$$\mathbf{Ax} + \underbrace{[-\mathbf{I} \quad \mathbf{R} \quad \mathbf{0}]}_{:=\,\mathbf{B}}\cdot\underbrace{\begin{bmatrix}\mathbf{v}\\ \mathbf{s}\\ \mathbf{s}'\end{bmatrix}}_{:=\,\bar{\mathbf{v}}} - \mathbf{l} = \mathbf{0}. \tag{8}$$

The representation given in Eq. (8) can be seen as a GHM. The method of least-squares leads to the normal equations system. The three main parts of LSC, the parameter estimation, the filtering of the signal and the prediction can be given as

$$\begin{bmatrix}\hat{\mathbf{v}}\\ \hat{\mathbf{s}}\\ \hat{\mathbf{s}}'\end{bmatrix} = \begin{bmatrix}-\mathbf{\Sigma_{vv}}\hat{\mathbf{k}}\\ \mathbf{\Sigma_{ss}}\mathbf{R}^{\mathrm{T}}\hat{\mathbf{k}}\\ \mathbf{\Sigma_{s's}}\mathbf{R}^{\mathrm{T}}\hat{\mathbf{k}}\end{bmatrix}, \tag{9}$$

where $\hat{\mathbf{k}}$ denotes the Lagrange multipliers, as follows

$$\hat{\mathbf{k}} = \underbrace{\mathbf{H}^{-1}(\mathbf{H} - \mathbf{A}(\mathbf{A}^{\mathrm{T}}\mathbf{H}^{-1}\mathbf{A})^{-1}\cdot\ \mathbf{A}^{\mathrm{T}})\mathbf{H}^{-1}}_{:=\ \mathbf{K}}\cdot\mathbf{l} \tag{10}$$

with $\mathbf{H} = \mathbf{\Sigma_{nn}} + \mathbf{R}\cdot\mathbf{\Sigma_{ss}}\cdot\mathbf{R}^{\mathrm{T}}$.

The VMCs of the quantities $\hat{\mathbf{v}}$, $\hat{\mathbf{s}}$, $\hat{\mathbf{s}}'$ and $\hat{\mathbf{k}}$ given in Eq. (9) and (10) can be derived using the law of propagation of uncertainty

$$\boldsymbol{\Sigma}_{\hat{\mathbf{k}}\hat{\mathbf{k}}} = \mathbf{K}^{\mathrm{T}} \cdot \boldsymbol{\Sigma}_{\mathbf{vv}} \cdot \mathbf{K}, \tag{11}$$
$$\boldsymbol{\Sigma}_{\hat{\mathbf{v}}\hat{\mathbf{v}}} = \boldsymbol{\Sigma}_{\mathbf{vv}} \cdot \boldsymbol{\Sigma}_{\hat{\mathbf{k}}\hat{\mathbf{k}}} \cdot \boldsymbol{\Sigma}_{\mathbf{vv}}, \tag{12}$$
$$\boldsymbol{\Sigma}_{\hat{\mathbf{s}}\hat{\mathbf{s}}} = \boldsymbol{\Sigma}_{\mathbf{ss}} \cdot \mathbf{R}^{\mathrm{T}} \cdot \boldsymbol{\Sigma}_{\hat{\mathbf{k}}\hat{\mathbf{k}}} \cdot \mathbf{R} \cdot \boldsymbol{\Sigma}_{\mathbf{ss}}, \tag{13}$$
$$\boldsymbol{\Sigma}_{\hat{\mathbf{s}}'\hat{\mathbf{s}}'} = \boldsymbol{\Sigma}_{\mathbf{s}'\mathbf{s}} \cdot \mathbf{R}^{\mathrm{T}} \cdot \boldsymbol{\Sigma}_{\hat{\mathbf{k}}\hat{\mathbf{k}}} \cdot \mathbf{R} \cdot \boldsymbol{\Sigma}_{\mathbf{ss}'}. \tag{14}$$

3. COLLOCATION APPROACH FOR PREDICTION AND FILTERING OF 3D TRAJECTORIES

3.1 Trend modelling

The circular motion of the ARP causes an sine oscillation which is seen as obvious trend in the measured data. Figure 2 depicts the time series of the north, east and up coordinates, which are referred to as time series from now on. The post-processing GNSS analysis yields the 3D coordinates which are transformed into a topocentric coordinate system (north, east, up) with origin in the reference station for further analyses. The MSS is assumed to be sufficiently orientated to the direction of gravity so the ARP's spatial circular motion is

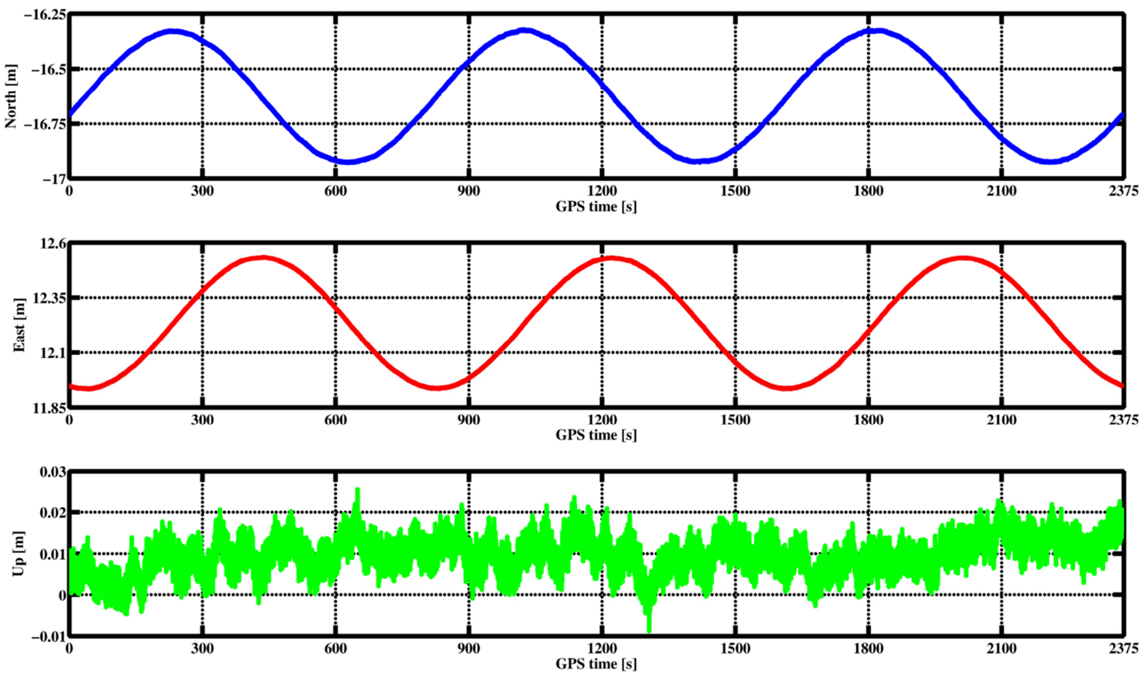

Figure 2: Time series of the north, east and up coordinates in a topocentric system. The GNSS data is acquired for $\approx 40\,min$ with the following GNSS equipment: *Javad TRE G3T Delta* Receiver operating with a data rate of 10 Hz connected to a *Javad GrAnt G3T* antenna. The nearby reference station is equipped with an identical receiver connected to a *Leica AR25* antenna. The reference station data was provided by the Institut für Erdmessung, Hannover.

located in a plane which is parallel to the north/east plane.

The mathematical description of this trajectory is considerably simplified by this transformation because in this case the circular motion affects only the north- and east-time series. Both time series clearly show the circular trend in form of a sine oscillation, while the up-time series is dominated by unsystematic variations around a value near zero (cf. Figure 2). Separate regression analyses are performed for the north- and east-time series to estimate the parameters of the sine oscillation. Thus, by the estimated sine parameters the circular trend can be removed and the starting angle of the circular motion can be approximated. After removing the circular trend, the time series show further regular systematics in form of a linear trend with varying degree. For this reason an additional linear regression is performed for the north- and east-time series as well as for the up-time series. The modelling of further trends, especially oscillations, is challenging. Therefore, the further modelling of oscillations has to be balanced against the estimation effort in the later collocation. This means, every trend, here in the representation of oscillations, which is removed beforehand the LSC has to be estimated during the collocation. In Harmening (2011) further oscillations are estimated for the time series but to keep the collocation approach simple only the linear trend and the sine oscillation trend are modelled.

3.2 Determination of appropriate covariance functions

The determination of appropriate covariance functions is performed according to Section 2.2: The first step is the computation of the empirical auto- and cross-covariance functions by means of Eq. (3) and Eq. (5). In the second step the estimated empirical auto- and cross-covariance functions are normalized according to Eq. (4) and Eq. (6). In the left part of Figure 3 the resulting autocorrelation functions are shown. The right part of Figure 3 depicts the cross-correlation functions.

The cross-correlation functions (Figure 3 right) show no obvious systematic behaviour and they only take small values in a range of ≈ -0.1 up to ≈ 0.2. Based on this findings, it is assumed that the cross-correlations can be neglected in the stochastic model.

All three autocorrelation functions (Figure 3 left) show a similar behaviour: They are characterised by a jump between the first and the second time step and descend distinctly slower to the function value $\widehat{K}(\Delta t) = 0$ afterwards. For $\Delta t > 250\ s$ the functions show small variations around $\widehat{K}(\Delta t) = 0$. This graphical behaviour suggests an exponential function being the analytical autocorrelation function. To estimate the analytical autocorrelation function by means of regression analysis two issues should be noted:

First, the definition of the autocorrelation function implies $\widehat{K}(0) = 1$. This condition is not considered within conventional regression analysis. This problem can be faced by introducing a restriction, which takes this additional requirement into account. The restriction is introduced by means of a two-stage adjustment: In the first stage the parameters are estimated which minimize the least-squares condition. In the second stage these parameters are adjusted by residuals which result from the restriction.

Second, the resulting exponential function does not sufficiently approximate the first values of the empirical function. Regarding the stochastic relationships among the data these first values provide the relevant information in terms of correlation of adjacent data. For that reason only a subset of the empirical values is taken into account to compute the analytical function. As a selection criterion the confidence interval $r_k(\Delta t)$ of the empirical correlation function is computed with the significance level $\alpha = 5\%$ (Jenkins and Watts, 1968)

$$r_k(\Delta t) = K_k(\Delta t) \pm 1{,}96 \cdot \sigma_K, \quad (15)$$

where σ_k denotes the standard deviation of the time series. The time difference Δt, where the lower bound of the confidence interval becomes smaller than zero, acts as a limit between those values which are included in the computation and those ones which are neglected. This is feasible because they do not differ significantly from zero at the 5% significance level.

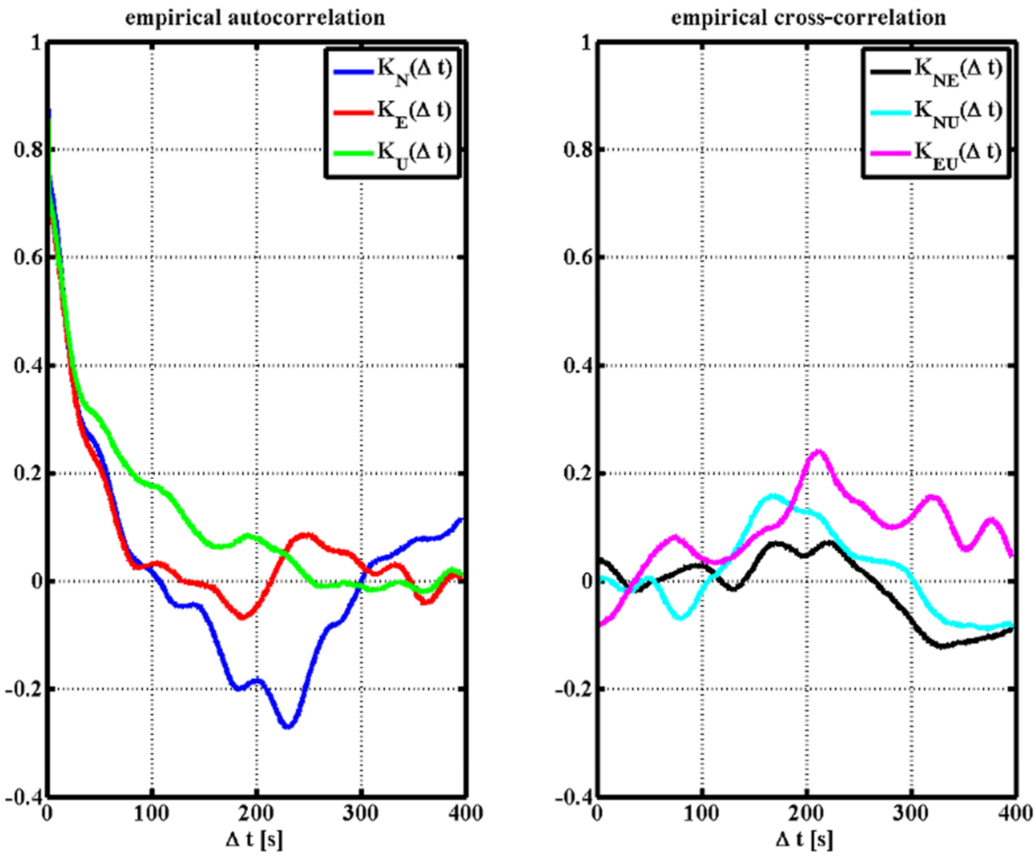

Figure 3: Left: Empirical autocorrelation functions for the north- (blue), east- (red) and up-time series (green). Right: Empirical cross-correlation functions for north-east- (black), north-up- (cyan) and east-up-time series (magenta).

In the estimation of the analytical functions the jump from the first to the second time step is considered as a problem to face. The restriction, that the autocorrelation function has to take the value $\hat{K}(0) = 1$, provides a good approximation of the estimated correlation function based on the empirical values. The piecewise estimation of more than one theoretical correlation function leads, unfortunately, to non-positive definite covariance matrices due to violation of the isotropy condition (Sansò and Schuh, 1987).

The left part of Figure 4 shows the empirical and analytical covariance functions for the east-times series. The corresponding empirical and analytical correlations functions are depicted in the right part of Figure 4. The analytical correlation function has been chosen from the family of exponential functions.

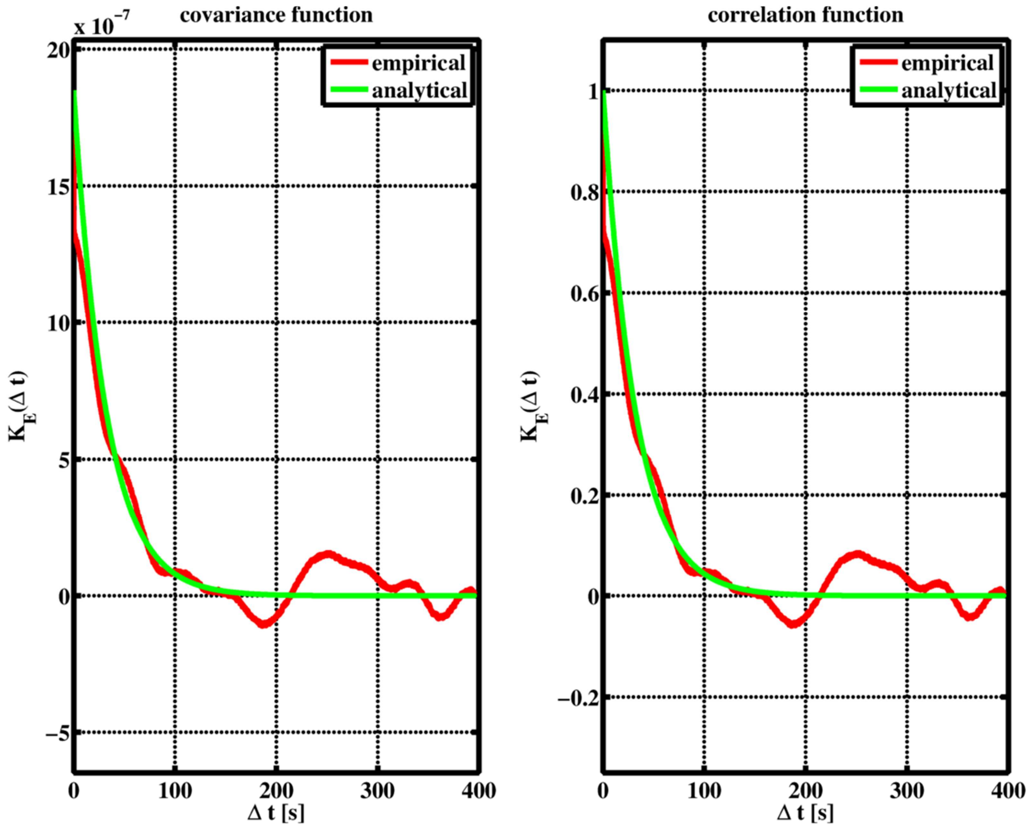

Figure 4: Empirical (red) and analytical (green) covariance function (left) and correlation function (right), respectively, of the east-times series.

3.3 Estimation, prediction and filtering of high resolution 3D GNSS trajectory

In this paper, we assume in Eq. (1) that **R** is equal to the identity matrix **I**, which express the functional relationship between observations and signal. This assumption simplifies the equations introduced in Section 2.3. It should be noted that the equations given in Section 2.3 are only valid for a linear relationship between parameters and observations.

Due to the nonlinear trend, which is caused by the circular motion, a linearization of the functional relationship is required. Therefore, on the one hand approximate parameters are needed and on the other hand an iterative solving of the normal equations has to be performed. It is worth noting, that the Lagrange multipliers (Eq. (10)) are computed by means of the truncated observation and parameter vectors $\Delta\mathbf{l}$ and $\Delta\mathbf{x}$, respectively

$$\hat{\mathbf{k}} = \mathbf{H}^{-1}(\Delta\mathbf{l} + \mathbf{A} * \Delta\hat{\mathbf{x}}). \tag{16}$$

Eq. (9) yields the estimated noise $\hat{\mathbf{v}}$ and signal $\hat{\mathbf{s}}$ based on $\hat{\mathbf{k}}$, given in Eq. (16).

The prediction of specific 3D positions $\mathbf{l}^*$ with respect to scan profiles out of the trajectory of the ARP (cf. Figure 2) requires the VCM $\mathbf{\Sigma_{s's}}$ given in Eq. (2). Therefore, in a first step the trend $\hat{\mathbf{l}}'$ is predicted using the estimated trend parameters. In a second step, the predicted signal $\hat{\mathbf{s}}'$ is determined. The predicted positions $\mathbf{l}^*$ are given by the sum of both: $\hat{\mathbf{l}}'$ and $\hat{\mathbf{s}}'$.

To evaluate the accuracies of the estimated quantities, the VCMs given in Eq. (11) – (14) are needed. Owing to the large scale of the VCMs (in some cases up to 25000×25000) the computation of the LSC becomes computationally expensive. Therefore, the computations are done using the RRZN cluster system.

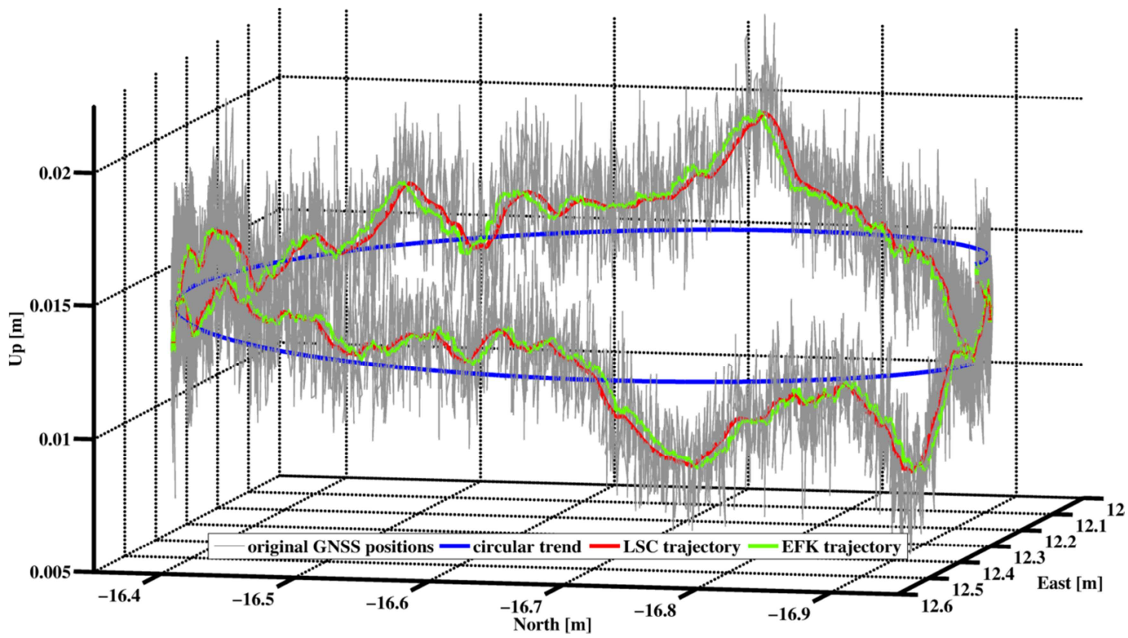

Figure 5: Filtering and prediction of 3D trajectory by means of LSC (red) and EKF (green).

Figure 5 shows the LSC versus the extended Kalman filter (EKF) results for the circular motion of the ARP with a diameter of $\approx 0.6\,m$. In gray the 3D positions obtained with the GNSS equipment are shown. The estimated trend is demonstrated in blue and the red line corresponds to the final predicted LSC result. The trajectory estimated by means of the EKF approach (Paffenholz et al., 2010) is drawn in green. Both methods: LSC and EKF, yield comparable results. Thus, no significant influence on derived transformation parameters by means of the trajectories are assumed. The filter effect is fulfilled in both approaches. An advantage of the EKF can be summarised in less computational effort on one side. On the other side, the EKF is capable for real-time applications due to the recursive estimation approach. The LSC trajectory differs from the trend by the predicted signal. In comparison to the measured trajectory the predicted one is much smoother, which means, that the influence of the measurement noise has been reduced by the filtering. Furthermore, irregular-systematic parts can be clearly seen in the predicted trajectory, which results from the consideration of the stochastic relationships.

4. CONLUSION AND OUTLOOK

This paper presents an LSC approach to solve the interpolation task in the frame of the briefly introduced direct geo-referencing approach (Paffenholz et al., 2010). In particular, the modelling of the trend and the determination of appropriate covariance functions is addressed. In order to filter and predict the 3D trajectory a trend elimination pre-step was needed. By means of -from trend eliminated- time series an empirical correlation function has been estimated. Whereas, the used time series are not completely free from deterministic effects, a good approximation by means of an exponential function has been reached. The positive definite correlation functions are used to build the VCMs needed for the stochastic model. The filter performance of the developed LSC approach shows good results. In addition, the prediction compared to the linear interpolation has taken the stochastic dependencies of adjacent time series values into account. The LSC approach has similar performance as the EKF approach for the filtering of the 3D trajectory. Thus, the use of the filtered trajectory in further processing steps to yield transformation parameters leads to comparable results. The main advantage of the EKF over the LSC approach is the real-time capability and less computational costs.

Topics of further investigations and research are identified in the field of trend modelling and the estimation of covariance functions. To evaluate the influence of remaining trend effects in the time series a more sophisticated collocation approach should be developed which accounts for more than one oscillation. Furthermore, the quality of the collocation approach and the estimated covariance functions, respectively, will be investigated by means of cross-validation (Cressie, 1993). The influence of the estimated covariance functions on the collocation results is another topic of future work which should be addressed according to Kotsakis (2007).

ACKNOWLEDGEMENT

The results presented here were (partially) carried out on the RRZN cluster system at the Leibniz Universität Hannover, Germany.

REFERENCES

Cressie, N. A. C. (1993): Statistics for spatial data. Rev. ed. New York, NY: Wiley (Wiley series in probability and mathematical statistics).

Harmening, C. (2011): Analysis of high resolution 3D trajectories by means of least-squares collocation. Bachelor thesis. Geodätisches Institut der Leibniz Universität Hannover.

Jenkins, G. M.; Watts, D. G. (1968): Spectral analysis and its applications. San Francisco: Holden-Day.

Koch, K.-R. (1999): Parameter estimation and hypothesis testing in linear models. 2. Ed. Berlin: Springer.

Kotsakis, C. (2007): Least-squares collocation with covariance matching constraints. In: *J Geod* 81 (10), pp. 661–677.

Moritz, H. (1962): Interpolation and prediction of gravity and their accuracy. OSU Rep 24.

Moritz, H. (1980): Advanced physical geodesy. Karlsruhe, Tunbridge, Eng: Wichmann; Abacus Press.

Neuner, H. (2008): Zur Modellierung und Analyse instationärer Deformationsprozesse. PhD thesis. München: DGK, Reihe C, 616.

Niemeier, W. (2008): Ausgleichungsrechnung. Statistische Auswertemethoden. 2. Ed.. Berlin: de Gruyter.

Paffenholz, J.-A.; Alkhatib, H.; Kutterer, H. (2010): Direct geo-referencing of a static terrestrial laser scanner. In: *JAG* 4 (3), 115-126.

Sansò, F.; Schuh, W.-D. (1987): Finite covariance functions. In: *Bull. Géodésique* 61, pp. 331–347.

Teunissen, P. J. G. (2006): Least-squares collocation with integer parameters. In: *Artificial Satellites* 41 (2), pp. 59–66.

Welsch, W.; Heunecke, O.; Kuhlmann, H. (2000): Auswertung geodätischer Überwachungsmessungen. In: M. Möser, G. Müller, H. Schlemmer und H. Werner (Eds.): Handbuch Ingenieurgeodäsie. Heidelberg: Wichmann.

BIOGRAPHICAL NOTES

Corinna Harmening received her Bachelor degree in Geodesy and Geoinformatics at the Leibniz Universität Hannover in 2011. Currently she is in the Master programme in Hanover.

Jens-André Paffenholz received his Dipl.-Ing. in Geodesy and Geoinformatics at the Leibniz Universität Hannover. Since 2006 he has been research assistant at the Geodetic Institute at the Leibniz Universität Hannover. His main research interests are: terrestrial laser scanning, industrial measurement systems, and process automation of measurement systems.

Dr. Hamza Alkhatib received his Dipl.-Ing. in Geodesy and Geoinformatics at the University of Karlsruhe in 2001 and his Ph.D. in Geodesy and Geoinformatics at the University of Bonn in 2007. Since 2007 he has been postdoctoral fellow at the Geodetic Institute at the Leibniz Universität Hannover. His main research interests are: Bayesian Statistics, Monte Carlo Simulation, Modeling of Uncertainty, Filtering in State Space Models.

CONTACTS

Corinna Harmening
Jens-André Paffenholz
Dr. Hamza Alkhatib
Geodätisches Institut
Leibniz Universität Hannover
Nienburger Str. 1, 30167 Hannover, Germany
Email: {paffenholz, alkhatib}@gih.uni-hannover.de