FIG Working Week 2012
Rome, Italy 6–10 May
Knowing to: Manage the territory, Protect the environment, Evaluate the cultural heritage

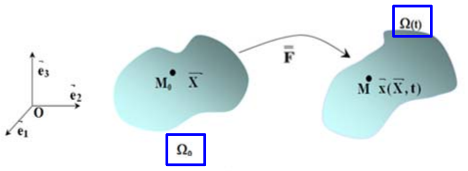
Centre of Space Techniques — Cts — Division of Space Geodesy

TRIANGULATION OF DELAUNAY : APPLICATION TO THE DEFORMATION MONITORING OF GEODETIC NETWORK BY USE OF STRAIN TENSORS

Session TS03F - Deformation Monitoring I

Belhadj Attaouia
b_attaouia@yahoo.fr

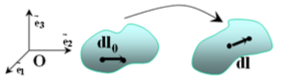
Deformation Study with Algebra Tensor



Continuous transformation / Tensor algebra

$$\bar{\bar{\varepsilon}} = \frac{1}{2}\left(\bar{\bar{F}}^T \bar{\bar{F}} - \bar{\bar{1}}\right)$$

$$\vec{dx} \cdot \vec{dx'} - \vec{dX} \cdot \vec{dX'} = \vec{dX}^T \left(\bar{\bar{F}}^T \bar{\bar{F}} - \bar{\bar{1}}\right) \vec{dX'} = 2\vec{dX}^T \bar{\bar{\varepsilon}}\, \vec{dX'}$$

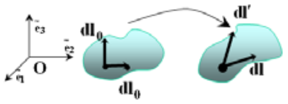


Translation

Elongation

$$\text{Si } \vec{dX} = \vec{dX'} = dl_0 \vec{e}_1$$

$$dl^2 - dl_0^2 = 2dl_0^2 \varepsilon_{11} \quad \text{soit } \varepsilon_{11} \approx \frac{dl - dl_0}{dl_0}$$



Angular Distortion

$$\text{Si } \vec{dX} = dl_0 \vec{e}_1 \text{ et } \vec{dX'} = dl_0 \vec{e}_2$$

$$dl\,dl' \cos\theta = 2dl_0^2 \varepsilon_{12} \quad \text{soit } 2\varepsilon_{12} \approx \cos\theta \sqrt{1+\varepsilon_{11}}\sqrt{1+\varepsilon_{22}}$$

Variation of Volume $\varepsilon_{ii} = \varepsilon_{xx} + \varepsilon_{yy}$

1

Small deformation hypothesis/ Linear Elasticity

$$dl'^2 = \sum_{i=1}^{3} dx'^2_i = \sum_{i=1}^{3} (dx_i + du_i)^2 = dl^2 + 2 \sum_{\substack{i=1...3 \\ j=1...3}} dx_i du_j + \sum_{i=1}^{3} du_i^2$$

Under Small Deformation hypothesis of a continuous homogeneous and elastic medium, the linearization of displacement field is verified

SDH
$\|\nabla \vec{u}\| << 1$

$$\varepsilon_{SDH} = \begin{bmatrix} \frac{\partial u_1}{\partial X_1} & .. \\ \frac{1}{2}\left(\frac{\partial u_1}{\partial X_2} + \frac{\partial u_2}{\partial X_1}\right) & \frac{\partial u_2}{\partial X_2} \end{bmatrix}$$

$$\delta u_i = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right)\delta x_k - \frac{1}{2}\left(\frac{\partial u_k}{\partial x_i} - \frac{\partial u_i}{\partial x_k}\right)\delta x_k$$

- **Symmetric Deformation**
- **Antisymmetric pure rotation**

2

Methods used in deformation study

Two main classes of solutions are used

1.Displacement vectors

Allow a good quality and legibility of final display movements. Require a definition of a reference system, fixed points…

2.Strain tensors

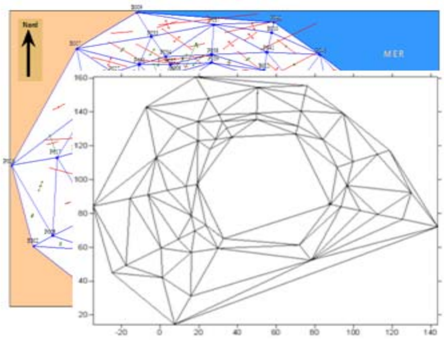
Need not input hypothesis. Do not require definition of a reference frame. Mainly gives the parameters of deformation

3

Problematic

- **Displacement vectors** : are not sufficient
- **Strain tensor :** quantifies deformation, but the results **differ** depending on the choice of elementary figures **(Michel.K, 2009)**

The solution proposed is the **study** of Strain Tensor on **Delaunay Triangulation** " **Uniqueness configuration** "



Study of Strain tensor on Delaunay Triangulation

Presentation plan

1. Motivation
2. Methodology
3. Application
4. Results
5. Conclusion

1.Motivation

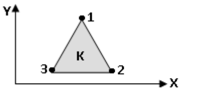
The objective of this study consists :

- To pass from **Displacement** information to **Deformation**
- To furnish the **deformation rate parameters**
- To Give a **confidence level** to these computed parameters
- **Display** the results of deformation on **Delaunay** triangulation

6

2.Methodology

Relies on computation of deformation tensors and significance degree of each finite element generated by Delaunay triangulation



Formalism of Triangular Finite Element

1. **Computation of displacement field**

$$X = (x, y)^T : Position$$

$$U(x_i, y_i) = (u_i, v_i)^T : Displacement$$

2. **Computation of deformation tensor** (**Akrour B,1990**)

$$\begin{bmatrix} dx_1 & dy_1 \\ dx_2 & dy_2 \\ dx_3 & dy_3 \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} * \begin{bmatrix} a_1 & a_4 \\ a_2 & a_5 \\ a_3 & a_6 \end{bmatrix}$$

$$\varepsilon = \nabla(U) = \begin{bmatrix} \varepsilon_{ux} & \varepsilon_{uy} \\ \varepsilon_{vx} & \varepsilon_{vy} \end{bmatrix}$$

3. **Deduce after tensor decomposition the primitives** (**Berber M,2006**)

Dilatation $: \lambda = \frac{1}{2}(\varepsilon_{ux} + \varepsilon_{vy})$

Shear $: \gamma = \frac{1}{2}\sqrt{(\varepsilon_{ux} - \varepsilon_{vy})^2 + (\varepsilon_{uy} + \varepsilon_{vx})^2}$

Differential rotation: $\delta\omega = \omega - \Omega = \frac{\varepsilon_{uy} - \varepsilon_{vx}}{2} - \Omega$

7

2.Methodology

4. **Provide significance of Deformation "Monte Carlo Method "** or measuring the influence of errors on the computation of tensor deformation
    - Proceed by Perturbation of measures **(Michel V, Person T, 2003)**
    - Calculation of **Deformability**, **Significance** ,**Significance level**

$$\begin{cases} \Sigma_{\gamma} = \dfrac{\gamma - \gamma_{déf}}{\gamma_{déf}} \\ \Sigma_{\lambda} = \dfrac{\lambda_{T} - \lambda_{Tdéf}}{\lambda_{déf}} \\ \Sigma_{\delta\omega} = \dfrac{|\delta\omega| - \delta\omega_{déf}}{\delta\omega_{déf}} \end{cases}$$

1. < 0 DNS

2. 0 < 1 DS&OD

3. > 0 DS

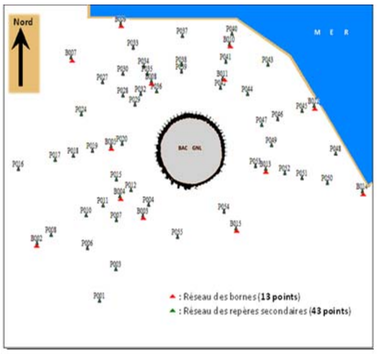
3.Application

The Auscultation concerns the monitoring of the site surrounding the Reservoir GL4Z Complex LNG at Arzew, Algeria

- **Natural reservoir** (1964) without any sealing barrier, only the soil freezing (-161 °) ensures its impermeability
- it presented Structural problems and natural, retired from service in 2007

3.Application

- **Coordinates** are expressed in the local geodetic reference and reported to the World Geodetic System"**WGS84** "
- **56 GPS** points obtained from two (02) campaigns observations (spaced six (06) years)
- **The data used** :
  - **2D network:** horizontal plane **E** (East horizontal component) and **N** (North horizontal component)
  - **1D network:** vertical direction **U** (vertical component or elevation)

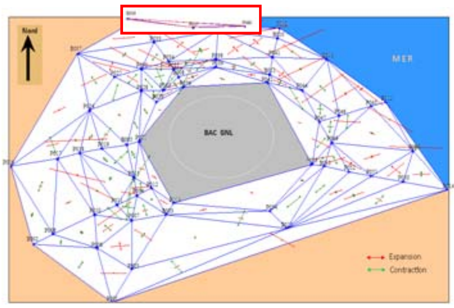


Auscultation Geodetic Network(2000-2006)

10

4. Result -2D - Deformation tensor

- The majority of mode deformation is an **Expansion** (North , North-East)
- Maximal Expansion
- Amplitude average of **Expansion** is higher than **contraction**

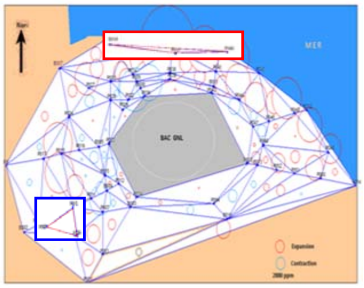


	Statistics of Main Contraintes (ppm: 10^{-6})			
	$_1$ (ppm)	**Triangles**	**$_2$ (ppm)**	**Triangles**
Min	-2631,9 *contraction*	P007, P011, B004	-12833,9 contraction	B008, P035, P039
Max	20700,4 *expansion*	P037, B009, P040	2700,8 expansion	P050, P051, P048
Mean	3502,3 *expansion*		-1522,0 *contraction*	

11

4.Result -2D - Dilatation (Expansion / Contraction)

- 61% of distribution is an **Expansion** (North ,North-East ,South -West)
- **Contraction** to the near of reservoir
- Maximal Expansion and contraction(North, North-South)



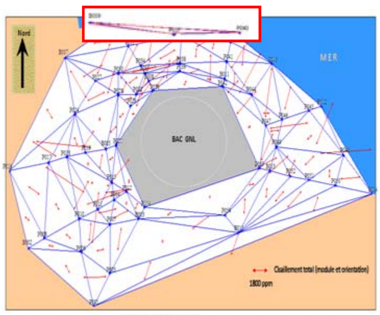
Dilatation

	Statistics of Dilatation (ppm: 10^{-6})			
	Contraction	Triangles	Expansion	Triangles
Min	77,1	P032,P035, B008	58,3	P054, P053, B013
Max	6589,7	P008, P010, P006	11480,8	P037, B009, P040
Mean	1604,9		2564,2	

12

4.Result -2D - Total shear

- **Highest modules**
  Northern ,North-East ,South -West
- **Similarity** with dilatation
  - Distribution (maximal value)
  - Average amplitude



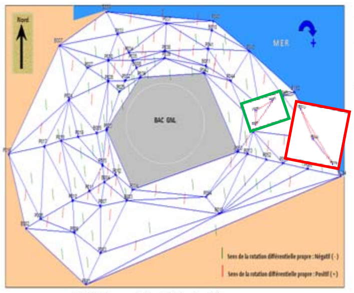
Total Shear

	Statistics of Total Shear(ppm: 10^{-6})	Triangles
Min	208,0	P011, P018, P015
Max	9219,6	P037, B009, P040
Mean	2512,2	

13

4.Result -2D - Differential rotation

- Most network areas had negative rotation (East-West)
- Average amplitude is West-East
- Maximal values (triangles in east extremity)

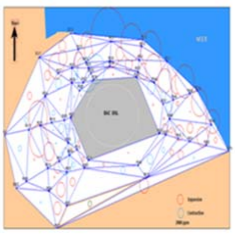


Differential Rotation

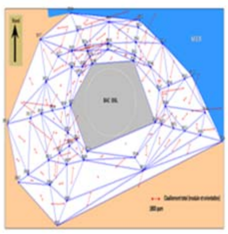
	Statistics of Rotation Différentielle Propre (degrés)			
	Sens (-)	**Triangles**	**Sens (+)**	**Triangles**
Min	0° 0024	P020, P028, P029	0° 0012	B010, P037,P040
Max	0° 2152	P049, P046, P045	0° 3865	P048, B012,B014
Mean	0° 0670		0° 0792	
***Rotation différentielle propre Moyenne du réseau : + 0° 0061** (West-East)*				

14

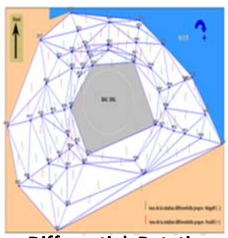
4.Result -2D - Significance of deformation tensor



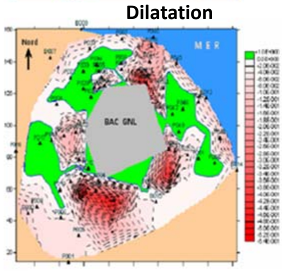
Dilatation



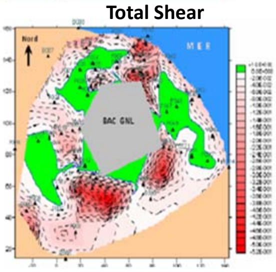
Total Shear



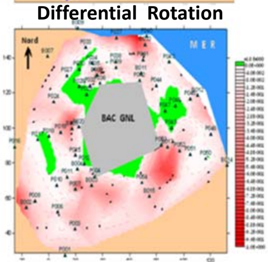
Differential Rotation



Significance of Dilatation



Significance of Total Shear



Significance of Rotation

DS&OD DNS

15

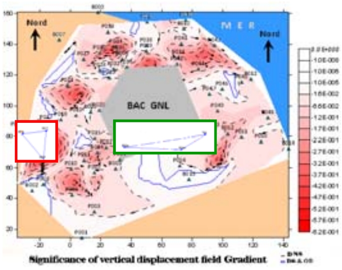
4.Result -1D – Vertical deformation, significance

Compaction/ Swelling

- 60% of compaction
- More important in South-West
- Average amplitude

Significance

- most parts of the study area did not deform significantly



	Statistics / Gradient of vertical displacement field (Unit elongation)			
	Compaction	**Triangles**	**Swelling**	**Triangles**
Min	0,003499	P035, P038, P039	0,000780	P049,P047, P046
Max	1,059294	P054, P004, P053	0,828593	P008, P016, P017
Mean	0,108850		0,147065	

16

5.Conclusion

- **Strain tensors and their primitives** , with mainly assessment of the **confidence degree of deformation** are important for estimation of real behavior of geodetic monitoring network
- Significance fits properly with deformation monitoring of geodetic networks in the same conditions, else it is essential to reconsider them
- **Delaunay triangulation** resolves the problem and permits the uniqueness of the exterior local deformation analysis

17

Conclusion

- However, this method is more appropriate to **good and dense geodetic networks geometry**
- Else , it is necessary to use other tools interpolations of field displacement to deepen deformation study

18